Random Oracles and Obfuscation

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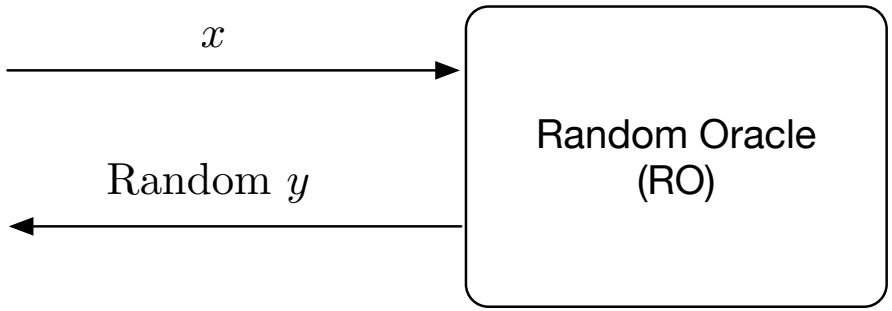
École Normale Supérieure

Summer School on Cryptology Crypto-CO

Random Oracles



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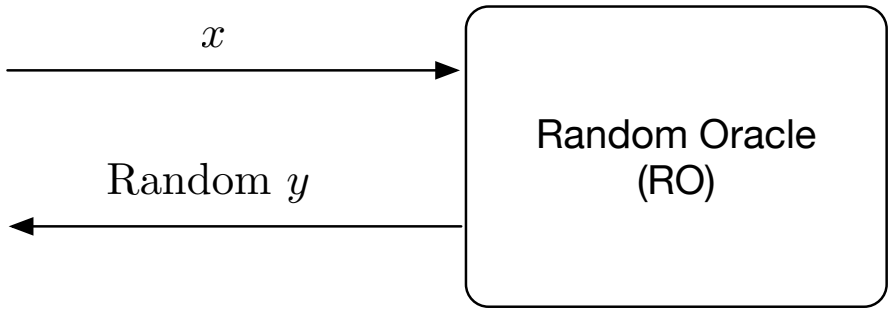

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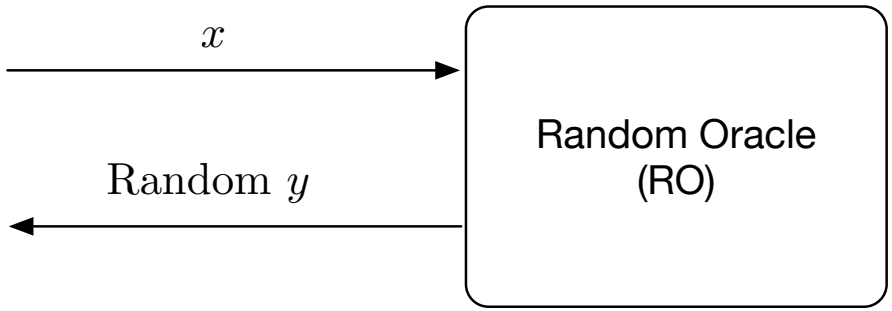

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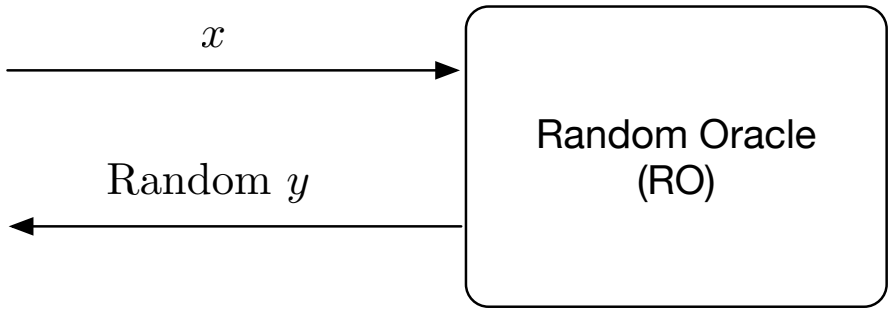

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- ROs enable the security proofs of a wide range of practical and strongly secure cryptosystems: encryption & signature schemes, key exchange, disk encryption, ...
- Standard-model counterparts are often less secure and/or less efficient.

RO Uninstantiability

- Reliance on ROs, although practical, is somewhat debatable:

  There are **uninstantiable** ROM schemes [CGH98].

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1. Interpret M as the **description** of a hash function Hash.
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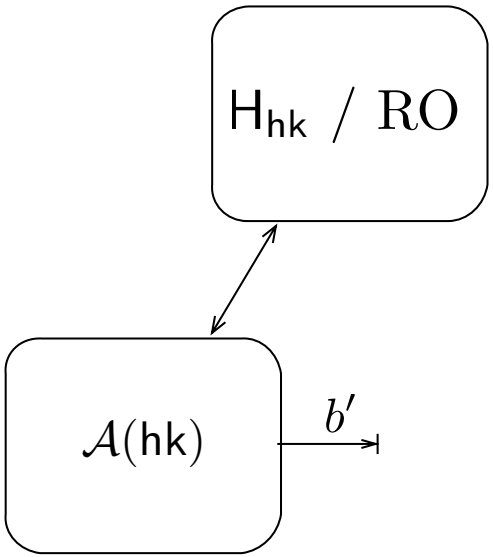
  3. Else return a normal/good encryption of M.
- Lack of a **definition** formalizing "RO-like" behavior.

(Very) Naïve Attempt at Modeling ROs

Call a hash function "IND-RO" if:



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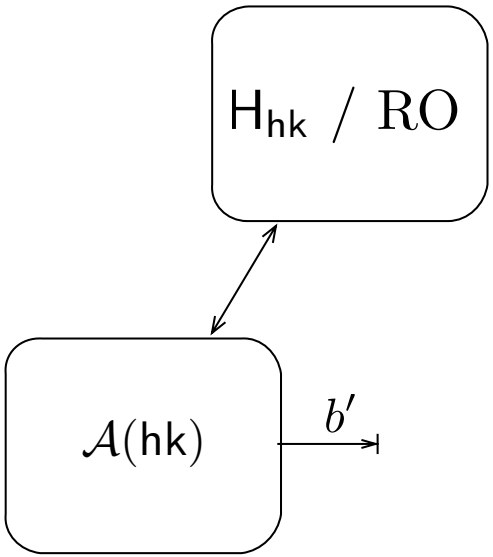

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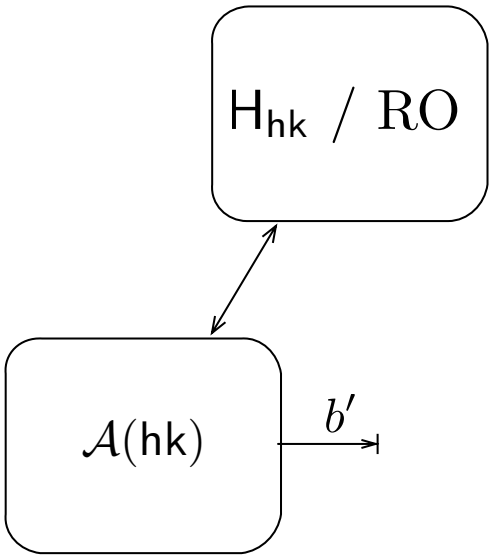

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Clearly uninstantiable:

$\mathcal{A}(\mathsf{hk})$: compute $\mathsf{H}_{\mathsf{hk}}(0)$ and compare to the oracle's reply.

But observe:

The adversary knows a full input, namely $(\mathsf{hk}, 0)$.

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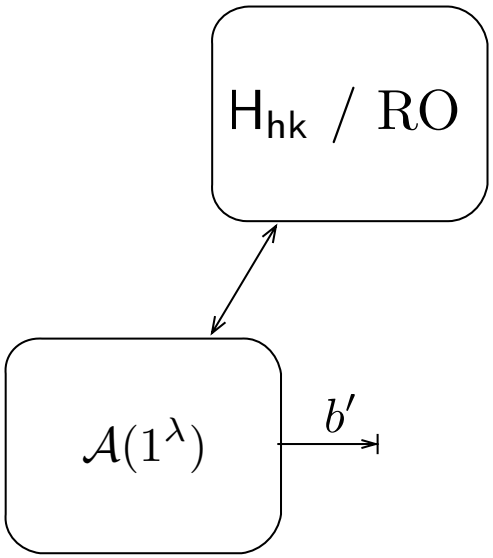

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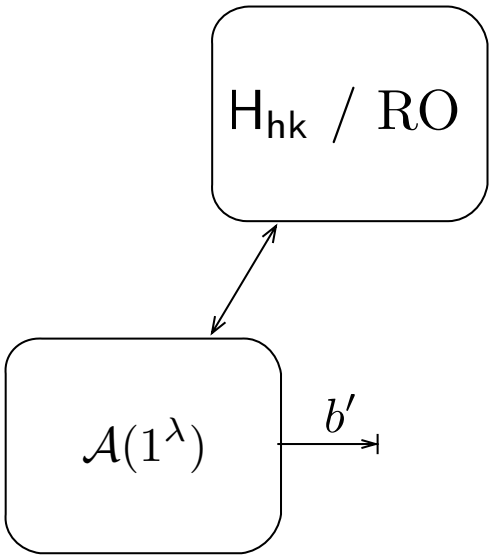

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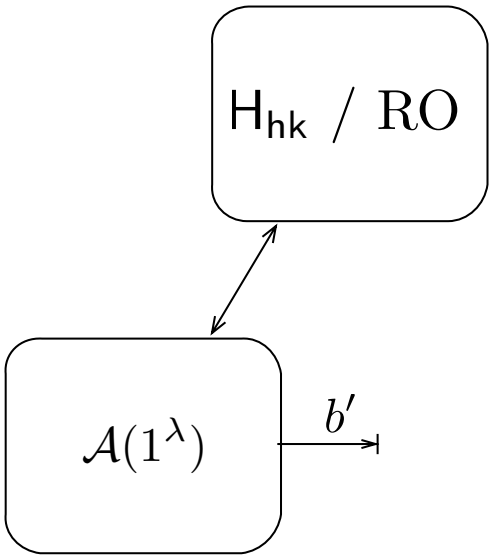

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First idea:

Split $\mathcal{A}$: one part gets hk and the other gets oracle access.

Modeling ROs via Split Adversaries

Call the two components of $\mathcal{A}$ the **source** $\mathcal{S}$ and the **distinguisher** $\mathcal{D}$:

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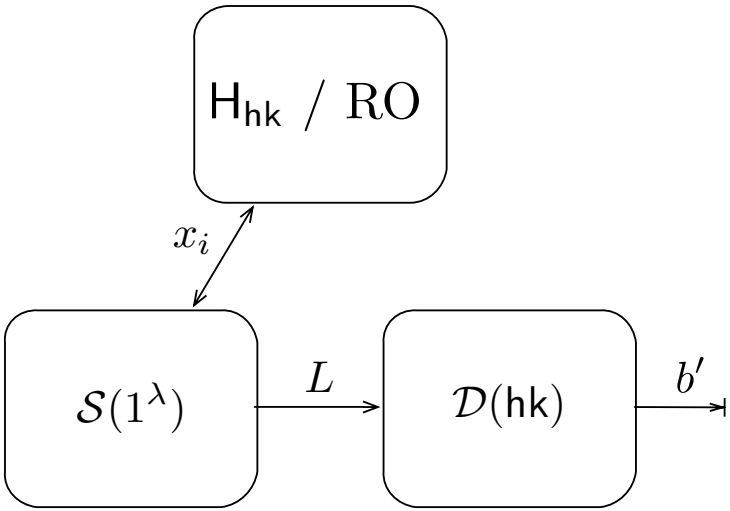

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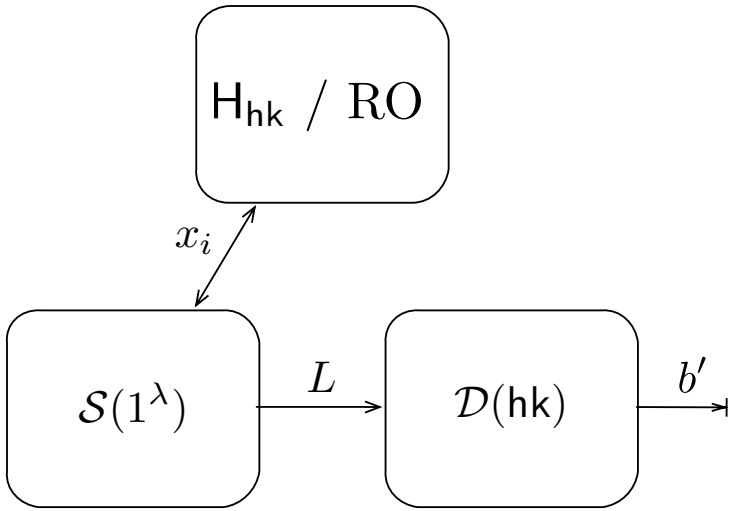

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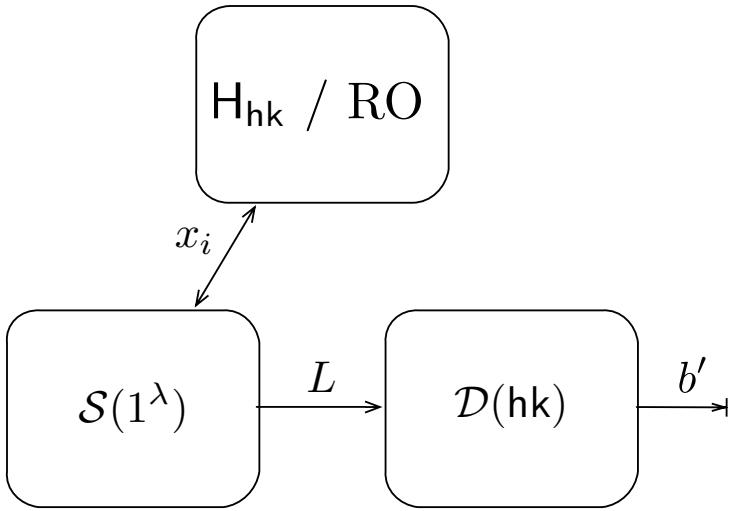

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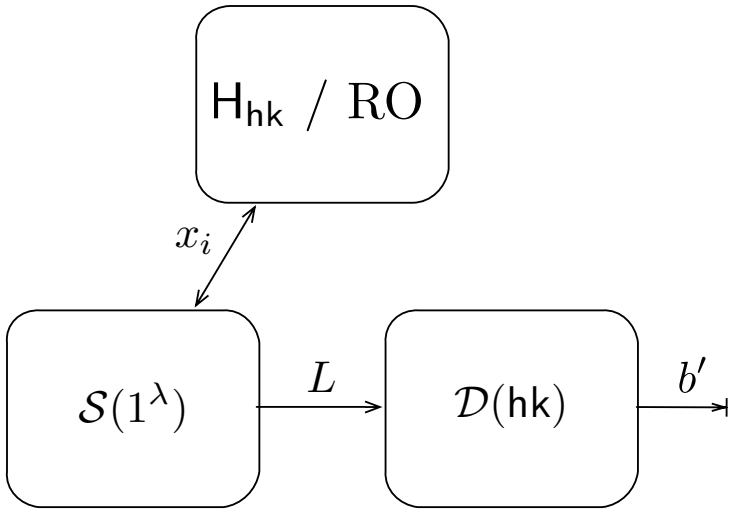

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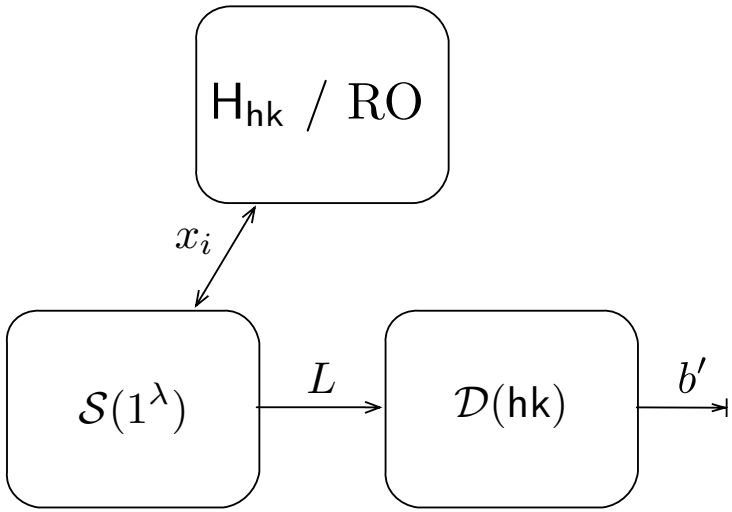
Second idea:

Restrict L: it must not leak any of $\mathcal{S}$'s queries.

Universal Computational Extractors (UCEs) [BHK13a]



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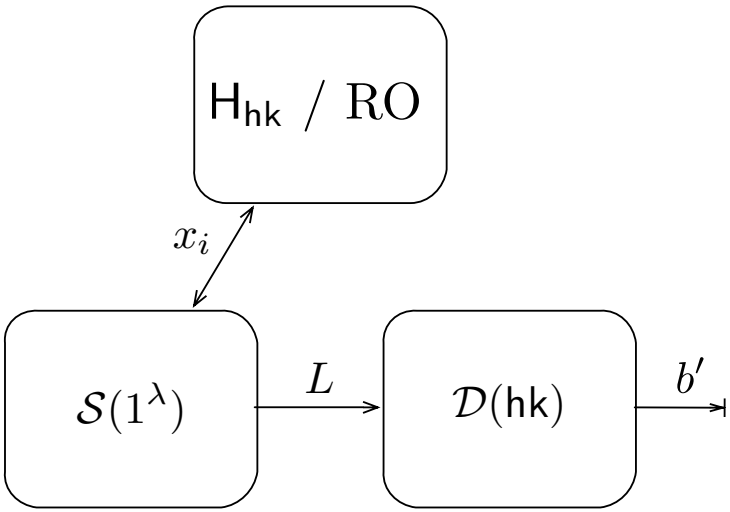

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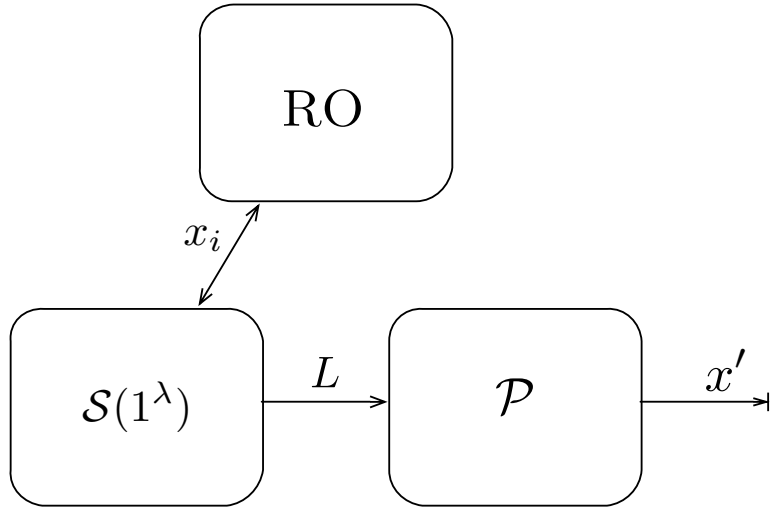

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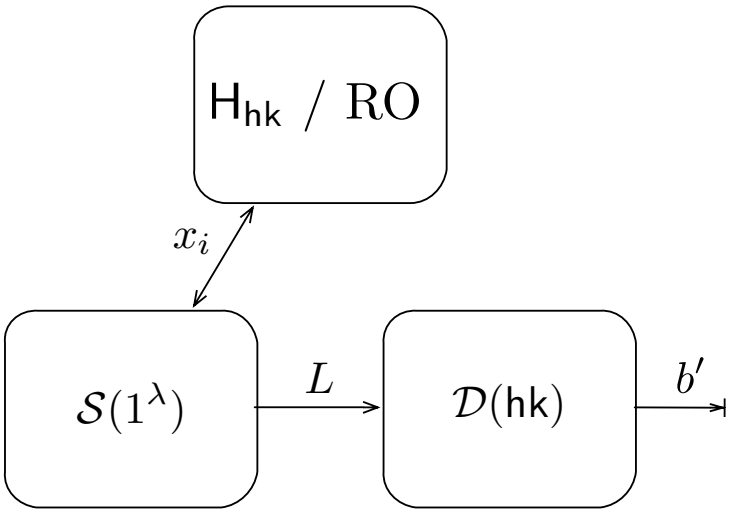

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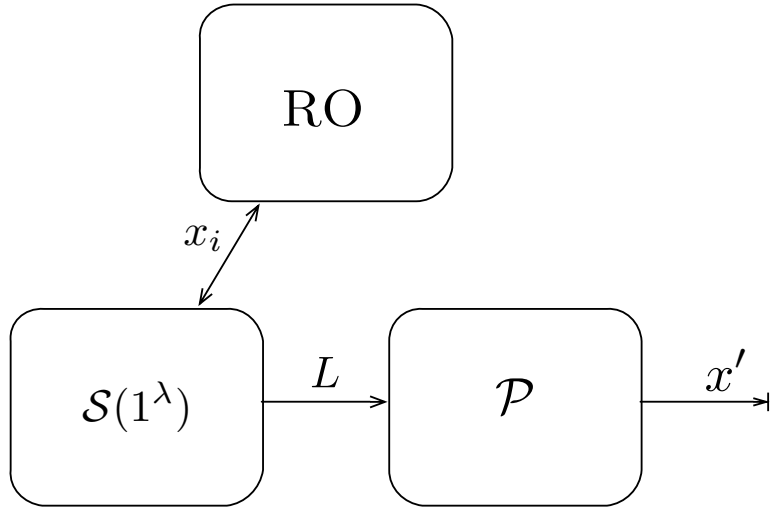

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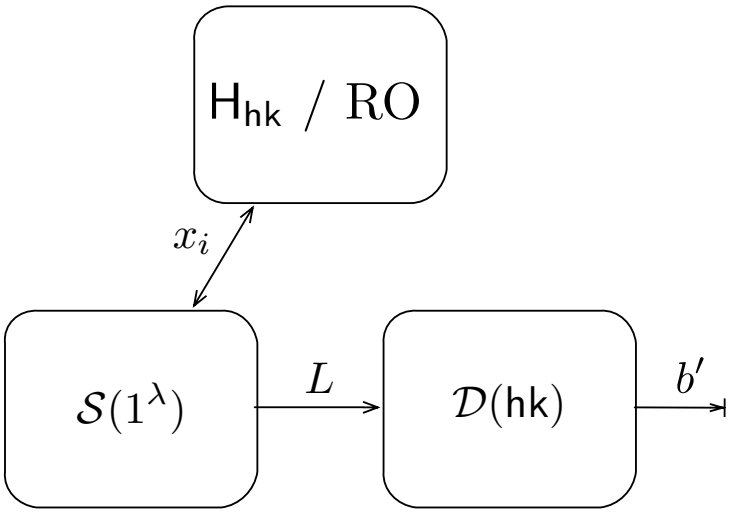

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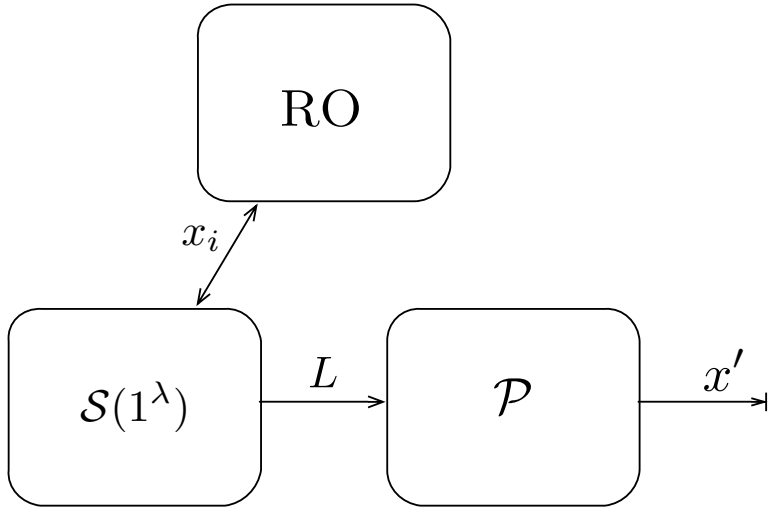

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H is **UCE1 secure** iff: $\mathbf{Adv}^{\mathrm{uce}}_{\mathsf{H},\mathcal{S},\mathcal{D}}(\lambda) \in \mathrm{Negl}$ for any **unpredictable** $\mathcal{S}$.

Applications of UCE [BHK13a]

UCE-secure hash functions can instantiate the RO in

- Deterministic PKEs
- RKA and KDM security
- Point-function obfuscation
- Message-locked encryption
- Proofs of storage
- Poly-many hard-core bits for any OWF
- OAEP, garbling schemes, ...

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Bottom line:

UCEs model many RO-like properties.

Is UCE Security Instantiable?

Suppose we could "strongly" obfuscate H:

- $\mathcal{S}$: Choose a random x. Query x to get y. Leak as L the values

$$y, \quad \mathsf{Obf}(\mathsf{H}(\cdot, x)) \ .$$

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Perhaps a different circuit and/or obfuscator might help?

Indistinguishability Obfuscation (iO)

Let C_0 and C_1 be two **functionally equivalent** circuits:

$$\forall x : C_0(x) = C_1(x) \ .$$

iO **security**: cannot efficiently distinguish obfuscations of such circuits:

$$\mathsf{iO}(C_0) \stackrel{c}{\approx} \mathsf{iO}(C_1) \ .$$

[GGH+13]: indistinguishability obfuscation for all poly-sized circuits from intractability assumptions related to multi-linear maps.

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Can we use iO to attack UCEs?

The iO Attack

$\mathcal{S}$: Leak an indistinguishably obfuscation of the Boolean circuit

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where x is random and y is oracle's response on x.

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Is $\mathcal{S}$ unpredictable?

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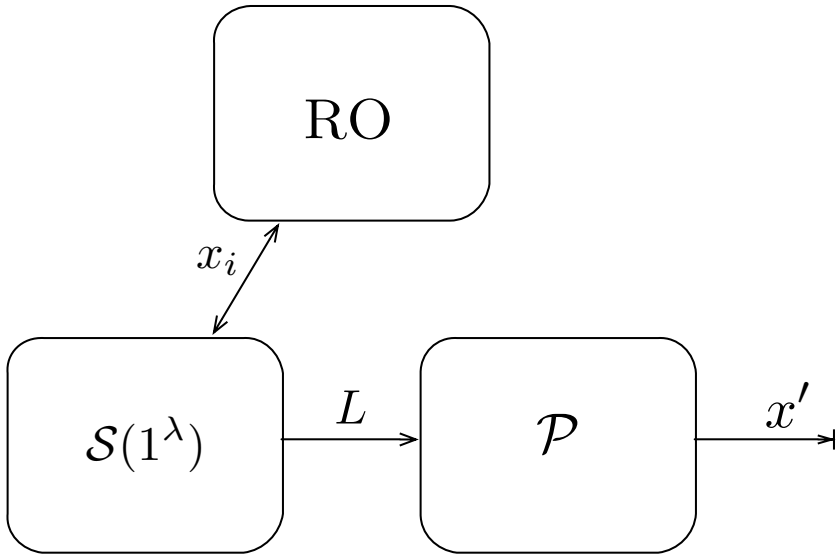

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with overwhelming probability. Now by the security of iO

$$\mathsf{iO}(\mathsf{H}(\cdot, x) \stackrel{?}{=} y) \quad \text{leaks no more than} \quad \mathsf{iO}(\mathsf{Zero})\ ,$$

and the latter is independent of x.

Dropping $|\mathsf{KSp}| \ll |\mathsf{Rng}|$

Consider instead iO-ing the circuit

$$\mathsf{H}(\cdot, x_1) \stackrel{?}{=} y_1 \ \wedge \ \cdots \ \wedge \ \mathsf{H}(\cdot, x_n) \stackrel{?}{=} y_n$$

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Theorem

Under the existence of iO, UCE security is uninstantiable.

Salvaging UCE: Statistical Unpredictability

iO is only **computationally** secure. Restrict the source class by

Allowing the **predictors** to run in **unbounded** time.

This raises the question: does **statistical iO** exist?

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Remark for researchers working on negative results:

Example of an impossibility result that is
used positively to define a security model.

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Does **not** hold for some other applications:

Reductions to UCE rely on L that only **computationally** hide x.

Final Thoughts

- UCEs bring added provable-security guarantees to practical protocols.
- Reliance on idealized objects,relpaced by UCEs.
- This work brings refinements to UCEs:
  - Computational UCE notion is uninstantiable under iO.
  - Our weaker statistical UCE is robust and still useful.
- iO attack extends to many other ROM schemes:
  - D-PKE, Hedged PKE, Fujisaki–Okamoto, KDM, MLEs, ...

Final Thoughts

- UCEs bring added provable-security guarantees to practical protocols.
- Reliance on idealized objects,relpaced by UCEs.
- This work brings refinements to UCEs:
  - Computational UCE notion is uninstantiable under iO.
  - Our weaker statistical UCE is robust and still useful.
- iO attack extends to many other ROM schemes:
  - D-PKE, Hedged PKE, Fujisaki–Okamoto, KDM, MLEs, ...

Thank you.