Introduction to Obfuscation

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An Obfuscated Code

```
                                                                    char
                                                                _3141592654[3141
        ],__3141[3141];_314159[31415],_3141[31415];main(){register char*
     _3_141,*_3_1415, *_3__1415; register int _314,_31415,__31415,*_31,
   _3_14159,__3_1415;*_3141592654=__31415=2,_3141592654[0][_3141592654
  -1]=1[__3141]=5;__3_1415=1;do{_3_14159=_314=0,__31415++;for( _31415
 =0;_31415<(3,14-4)*__31415;_31415++)_31415[_3141]=_314159[_31415]= -
1;_3141[*_314159=_3_14159]=_314;_3_141=_3141592654+__3_1415;_3_1415=
__3_1415    +__3141;for              (_31415 = 3141-
          __3_1415  ;                _31415;_31415--
          ,_3_141 ++,                _3_1415++){_314
          +=_314<<2 ;                _314<<=1;_314+=
         *_3_1415;_31                 =_314159+_314;
         if(!(*_31+1)                 )* _31 =_314 /
         __31415,_314                 [_3141]=_314 %
         __31415 ;* (                 _3__1415=_3_141
        )+= *_3_1415                   = *_31;while(*
        _3__1415 >=                    31415/3141 ) *
        _3__1415+= -                   10,(*--_3__1415
       )++;_314=_314                   [_3141]; if ( !
       _3_14159 && *                   _3_1415)_3_14159
       =1,__3_1415 =                   3141-_31415;}if(
       _314+(__31415                    >>1)>=__31415 )
       while ( ++ *                     _3_141==3141/314
      )*_3_141--=0                      ;}while(_3_14159
      ) ; { char *                      __3_14= "3.1415";
      write((3,1),                      (--*__3_14,__3_14
      ),(_3_14159                        ++,++_3_14159))+
     3.1415926; }                        for ( _31415 = 1;
    _31415<3141-                         1;_31415++)write(
   31415% 314-(                          3,14),_3141592654[
  _31415    ] +                         "0123456789","314"
  [ 3]+1)-_314;                         puts((*_3141592654=0
,_3141592654))                           ;_314= *"3.141592";}
```

What's Obfuscation?

Obfuscation is the task of **making programs unintelligible.**

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Such that:

- Programs P and $\overline{P}$ have the same functionality.
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In a sense, we want to "encrypt programs" (rather than messages).

Why Look at Obfuscators?

With a general-purpose obfuscator:

- None can we run arbitrary programs
- while **hiding secrets** within those programs.
- And have guarantees that the program code does not leak any unintentional information.

Potential Applications

- **Software protection**: Publish software resilient to reverse-engineering:

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$$\text{Distribute } pk := \mathsf{Obf}(\mathsf{Enc}(k, \cdot))$$

A circuit that allows encryption under k without revealing k.

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  A circuit that allows encryption under k without revealing k.

- **Hash Functions**: Public random functions:

$$\mathsf{H}(\cdot) := \mathsf{Obf}(\mathsf{AES}(k, \cdot))$$

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A **homomorphic encryption** scheme allows

Computing $\mathsf{Enc}(k, f(m))$ from $\mathsf{Enc}(k, m)$ **without** k.

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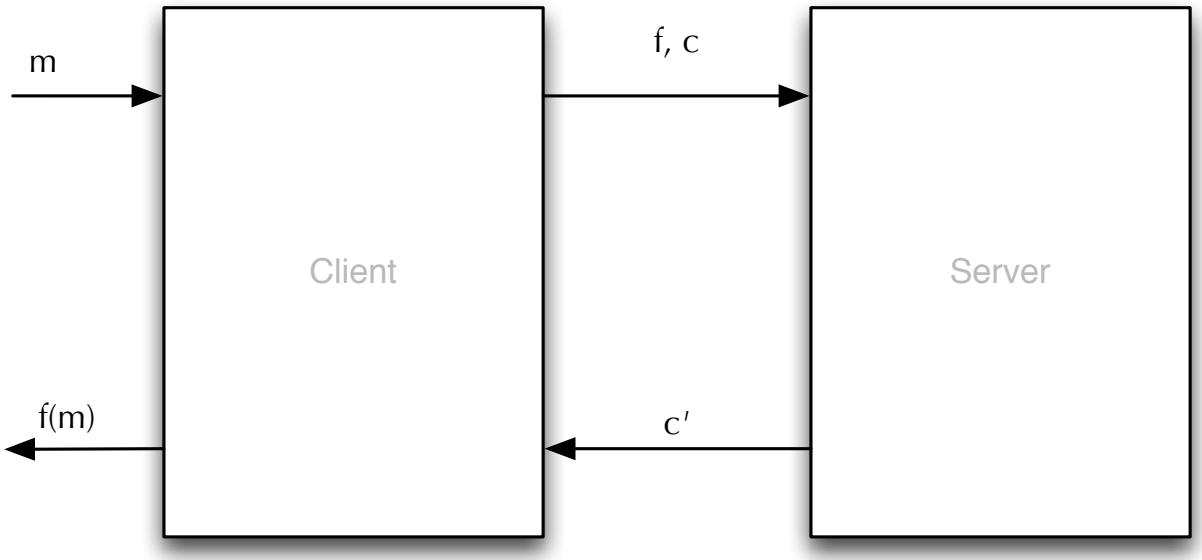
We can use this to **privately outsource computation**:



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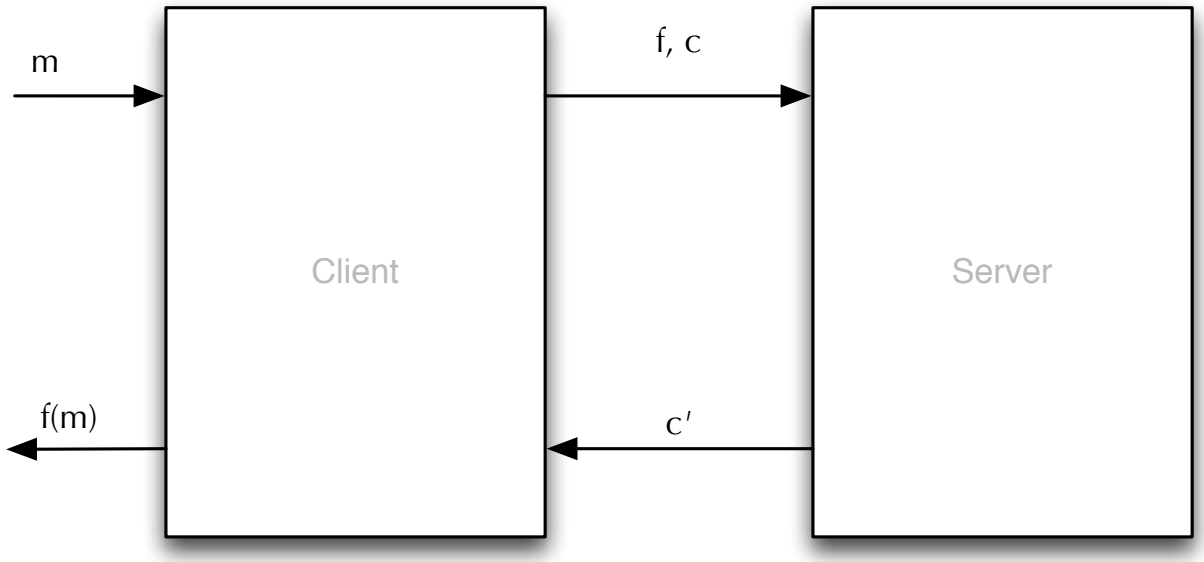

Solution: Obfuscate "decrypt, apply f, re-encrypt":

$$\text{Publish } \mathsf{Eval}(f, c) := \mathsf{Obf}\Big(\mathsf{Enc}\big(k, f\big(\mathsf{Dec}(k, c)\big)\big) \Big)$$

Functional Encryption

Traditional Public-Key Encryption is **all-or-nothing**:

> You either know sk and can recover m fully from $\mathsf{Enc}(pk, m)$
> or you don't know it, and you don't get any information on m.

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Functional Encryption extends PKEs to a **fine-grained** setting:

> For each function f an associated secret key sk_f allows
> recovering $f(m)$ from $\mathsf{Enc}(pk, m)$ and nothing more.

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(**Note:** Difference with hom. enc. is that output is in the clear.)

A Simple Application

Consider the program:

$$C[pwd](x) := \begin{cases} \text{Success} & \text{if } x = pwd; \\ \text{Access denied} & \text{otherwise.} \end{cases}$$

Useful for log-in applications.

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Obfuscation ensures pwd remains hidden given a code for $C[pwd]$.

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- Watermarking software
- ...

Obfuscation is an extremely powerful tool:

A "crypto-complete" primitive, a "central hub" for crypto, ...

Syntax and Correctness

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Efficiency: $\mathsf{Obf}(C)$ compiles C to a program $\overline{\mathsf{C}}$ that is polynomially slower than C.

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$$\mathcal{A}(\mathsf{Obf}(C)) \quad \overset{c}{\approx} \quad \mathcal{A}^{C} \ .$$

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VBB Security: For all efficient $\mathcal{A}$ there is an efficient **simulator** $\mathcal{S}$ such that for all circuits C:

$$\mathcal{A}(\mathsf{Obf}(C)) \quad \stackrel{c}{\approx} \quad \mathcal{S}^C \ .$$

[BGI+01]

On the (Im)possibility of Obfuscating Programs*

Boaz Barak† Oded Goldreich‡ Russell Impagliazzo§ Steven Rudich¶
Amit Sahai‖ Salil Vadhan** Ke Yang††

July 29, 2010

Abstract

Informally, an *obfuscator* $\mathcal{O}$ is an (efficient, probabilistic) "compiler" that takes as input a program (or circuit) P and produces a new program $\mathcal{O}(P)$ that has the same functionality as P yet is "unintelligible" in some sense. Obfuscators, if they exist, would have a wide variety of cryptographic and complexity-theoretic applications, ranging from software protection to homomorphic encryption to complexity-theoretic analogues of Rice's theorem. Most of these applications are based on an interpretation of the "unintelligibility" condition in obfuscation as meaning that $\mathcal{O}(P)$ is a "virtual black box," in the sense that anything one can efficiently compute given $\mathcal{O}(P)$, one could also efficiently compute given oracle access to P.

In this work, we initiate a theoretical investigation of obfuscation. Our main result is that, even under very weak formalizations of the above intuition, obfuscation is impossible. We prove this by constructing a family of efficient programs $\mathcal{P}$ that are *unobfuscatable* in the sense that (a) given *any* efficient program P' that computes the same function as a program $P \in \mathcal{P}$, the "source code" P can be efficiently reconstructed, yet (b) given *oracle access* to a (randomly selected) program $P \in \mathcal{P}$, no efficient algorithm can reconstruct P (or even distinguish a certain bit in the code from random) except with negligible probability.

We extend our impossibility result in a number of ways, including even obfuscators that

Feasibility

Let's start with

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Let f be a one-way permutation. Define $\mathsf{Obf}(C[x^*](x))$ as:

- Let $y^* := \pi(x^*)$
- Return the program:

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Correctness: $\overline{C}$ preserves functionality as π is a permutation.
Security: Intuition: Think of π as being ideal.

Bad News

$\mathcal{A}$ can run the obfuscated code on itself, but $\mathcal{S}$ cannot.

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Using other techniques one can show:

VBB obfuscation is **impossible** for wide classes of circuits,
including many cryptographically interesting ones.

This leads to the question:

Are there weaker (or incomparable) notions of security
that are both feasible and useful?

Breakthrough Result of 2013

Candidate Indistinguishability Obfuscation and Functional Encryption for all circuits

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July 21, 2013

Abstract

In this work, we study *indistinguishability obfuscation* and *functional encryption* for general circuits:

Indistinguishability obfuscation requires that given any two equivalent circuits C_0 and C_1 of similar size, the obfuscations of C_0 and C_1 should be computationally indistinguishable.

In functional encryption, ciphertexts encrypt inputs x and keys are issued for circuits C. Using the key SK_C to decrypt a ciphertext $\mathrm{CT}_x = \mathsf{Enc}(x)$, yields the value $C(x)$ but does not reveal anything else about x. Furthermore, no collusion of secret key holders should be able to learn anything more than the union of what they can each learn individually.

We give constructions for indistinguishability obfuscation and functional encryption that supports all polynomial-size circuits. We accomplish this goal in three steps:

- We describe a candidate construction for indistinguishability obfuscation for $\mathbf{NC}^1$ circuits. The security of this construction is based on a new algebraic hardness assumption. The candidate and assumption use a simplified variant of multilinear maps, which we call *Multilinear Jigsaw Puzzles*.
- We show how to use indistinguishability obfuscation for $\mathbf{NC}^1$ together with Fully Homomorphic Encryption (with decryption in $\mathbf{NC}^1$) to achieve indistinguishability obfuscation for all circuits.
- Finally, we show how to use indistinguishability obfuscation for circuits, public-key encryption, and non-interactive zero knowledge to achieve functional encryption for all circuits. The functional encryption scheme we construct also enjoys succinct ciphertexts, which enables several other applications.

Indistinguishability Obfuscation (iO)

Call two circuits C_0 and C_1 **functionally equivalent** if

$$(\forall x) \ : \quad C_0(x) = C_1(x) \ .$$

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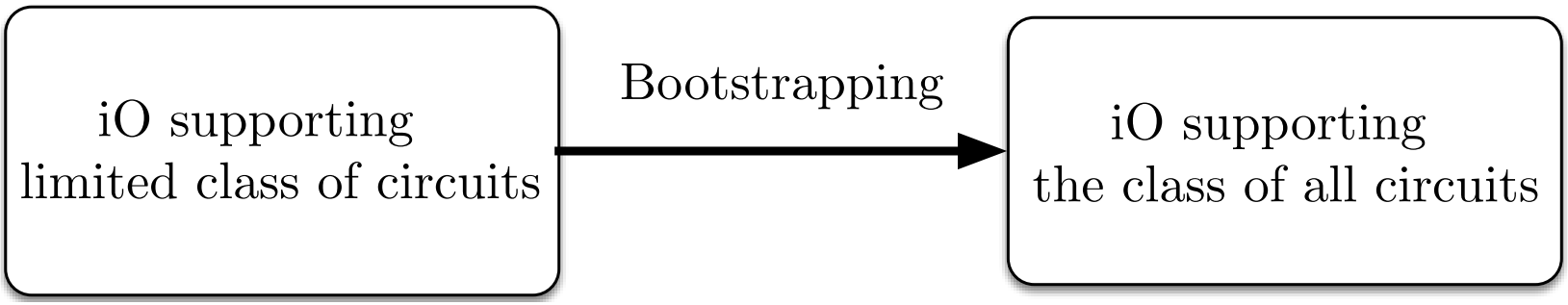
For instance, the obfuscations of

$$C_0(x, y) := (x + y)^2 \quad \text{and} \quad C_1(x, y) := (x^2 + y^2 + 2xy) \ .$$

should look indistinguishable.

iO Construction



iO for $\mathbf{C}_{\text{small}}$

Recall bilinear maps (aka. pairings):

$$e : \mathbb{G} \times \mathbb{G} \longrightarrow \mathbb{G}_T$$

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For iO we need k-linear maps:

$$e : \mathbb{G} \times \cdots \times \mathbb{G} \longrightarrow \mathbb{G}_T$$
$$e(g^{a_1}, \ldots, g^{a_k}) = e(g, \ldots, g)^{a_1 \cdots a_k}$$

- Allow computing polynomials of degree $\leq k$ in the exponent.
- Checking if the output was zero or one.
- Multi-linear map security $\implies$ iO security

Fully Homomorphic Encryption

1. $\mathsf{Gen}(\lambda)$ outputs a key pair (sk, pk)
2. $\mathsf{Enc}(pk, m)$ outputs a ciphertext c.
3. $\mathsf{Eval}(pk, f, c)$ outputs an evaluated ciphertext c.
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Security: Identical to IND-CPA security.

$$(pk, \mathsf{Enc}(pk, m_0)) \overset{c}{\approx} (pk, \mathsf{Enc}(pk, m_1))$$

The Bootstrapping Step

Intuition:

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Now proceed as follows:

- Run $\mathsf{Eval}(pk, \mathrm{Universal}(\cdot, x), \mathsf{Enc}(pk, C))$ to get c'. Ciphertext c' now encrypts $C(x)$.
- Run $\mathsf{iO}(\mathsf{Dec}(sk, \cdot))$ on c' to decrypt it and recover $C(x)$.

An Application of iO

The Symmetric-to-Asymmetric Conversion

SE-to-PKE Conversion

Recall one consequence of VBB obfuscation was:

We can convert a symmetric encryption to a public-key one.

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- Yes, but we now need a special type of Enc.

Tools

Some more foundational primitives.

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Syntax: A PRG is deterministic length-increasing function:

$$\mathsf{PRG} : \{0,1\}^n \longrightarrow \{0,1\}^{n+s} \ .$$

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- Prove PRG $\implies$ OWF.
- The other direction, OWF $\implies$ PRG, is much harder. One of the main classical results in crypto.

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$$\mathcal{A}^{\mathsf{PRF}(k,\cdot)} \quad \overset{c}{\approx} \quad \mathcal{A}^{\mathsf{RO}(\cdot)}$$

Here RO is a **Random Oracle** (aka. random function):

- On each input x it returns a truly random value $y := \mathsf{RO}(x)$
- On repeat inputs it's consistent and repeats outputs.

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- On repeat inputs it's consistent and repeats outputs.

Prove: PRF $\implies$ PRG. (Use PRF a few times.)
Fact: The GGM construction shows: PRG $\implies$ PRF.

Summary

PRG : a function that stretch random strings form n bits to $m > n$ bits.

PRF : a function that behaves like a random functions as long as its key remains hidden.

The Conversion

The Symmetric-to-Asymmetric Conversion

A Basic Symmetric Encryption

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Prove: This scheme is IND-CPA secure if the PRF is secure.
Intuition: XORing with PRF acts as a one-time pad.

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- We need to use a special type of PRF in place of a standard PRF.

Exercise: Prove this scheme is still IND-CPA secure.
Intuition: PRG is secure, so randomness r looks random. The rest is as before.

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Correctness: Follows form the correctness of SE and the correctness of iO.
Security: Takes 2/3 more slides.

Security Proof

Theorem

For any two messages m_0 and m_1:

$$(pk, \mathsf{Enc}(pk, m_0)) \overset{c}{\approx} (pk, \mathsf{Enc}(pk, m_1))$$

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Proof.

Gm	pk	c_1	c_2	**Why?**
0	$\mathsf{iO}(\mathsf{Enc}(k, \cdot; \cdot))$	$r^* := \mathsf{PRG}(s)$	$m_b \oplus \mathsf{PRF}(k, r^*)$	
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We want to show for all m, s and a random r^*:

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This holds with overwhelming probability as

- PRG has a sparse image in $\{0,1\}^{2\lambda}$.
- r^* was chosen randomly in $\{0,1\}^{2\lambda}$.

Other Applications of iO

- Deniable Encryption
- Hierarchical/Delegatable Functional Encryption
- Removing Reliance on Random Oracles
- Witness Encryption
- Multilinear Maps
- Zero-Knowledge Proofs
- ...

Further Reading

2001 (Im)possibility of Obfuscating Programs. (Barak et al.)

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